Theorem 1 (Dominated convergence of Lebesgue) *Assume that g is an integrable function defined on the measurable set E and that $(f_n)_{n\in\mathbb{N}}$ is a sequence of measurable functions so that $|f_n| \leq g$. If f is a function so that $f_n \to f$ almost everywhere then*

$$\lim_{n\to\infty} \int f_n = \int f.$$

Proof: The function $g - f_n$ is *non-negative* and thus from Fatou lemma we have that $\int (g - f) \leq \liminf \int (g - f_n)$. Since $|f| \leq g$ and $|f_n| \leq g$ the functions f and f_n are integrable and we have

Figure 1: Caption in Sans fonts

$$\int g - \int f \leq \int g - \limsup \int f_n,$$

so

$$\int f \geq \limsup \int f_n.$$